Exercise 3: Segmentation, Optical Flow and Nonlocal Operators

Exercise due on 29.1.2018

Please send a PDF file containing a full solution (analytic and experimental results) and the Matlab files folder to **Tal Feld, stmfeld@gmail.com.** The subject of the mail should be **"Exercise 3"**. In the mail please write the full names + ID of the participants.

1 Analytic exercises (30 points)

1. *Spectral TV:*

    (a) Construct an ideal TV high-pass-filter $HPF(t)$ which is a function of $u(t)$, t, $p(u(t))$ and f (but not of $\phi(t)$). That is, for $f(x)$ an eigenfunction with eigenvalue λ (Notes 5, Eq. (9)), with mean value $\bar{f} = 0$, we get $u(t; x) = f(x)$ for $t > 1/\lambda$ and 0 otherwise. Prove your claim.

    (b) Let $\hat{S}^2(t) = \langle f, \phi(t) \rangle$. Show a Parseval-type rule: $\int_0^\infty \hat{S}^2(t)dt = \|f\|_{L^2}^2$ for f with mean value 0.

2. *Nonlocal calculus:*

    (a) Show the divergence theorem, Eq. (10), Notes 10.

    (b) Creative: For nonsymmetric weights $w(x, y) \neq w(y, x)$, suggest a nonlocal gradient and divergence which are adjoint, that is obey Eq. (9), Notes 10. What is the nonlocal Laplacian in this case?
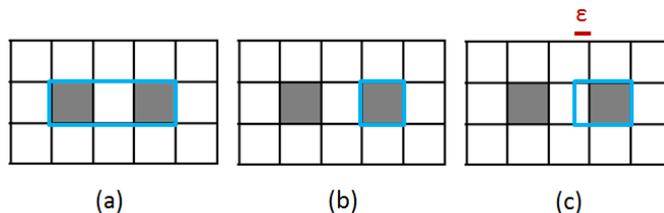
Figure 1: Chan-Vese segmentation - curve configurations.

3. **Chan-Vese (12 points).** The question is based on the Chan-Vese energy, Eq. (3), Notes 8. In Fig. 1 assume the squares are each of size 1×1. The white parts are of value 1 and the 2 dark squares are of value 0. The (blue) curve is C. Assume the curve falls exactly on the grid lines. For simplicity, assume the curve can only be in straight lines on the grid (ignore smoothing of the corners etc.).

   (a) Assume c_1 and c_2 reach their optimal value for each curve. Compute E_{CV} for cases (a) and (b) in Fig. 1. Explain your result, which values of μ would you suggest to use?

   (b) For the curve in (b) the left side is expended by ε outside the object in a straight line, as shown in (c). Compute the new energy of E_{CV}. Can a curve in the form shown in (b) evolve by minimizing the E_{CV} energy to be the curve as shown in (a)? Explain.

   (c) Creative: suggest a small variation on the E_{CV} energy (Eq. (3)) such that for this case the preferred energy would be more likely to include both dark objects in the curve compared to the current energy (with a final curve as in (a) or as 2 separate objects).

2 Matlab experiments (70 points)

1. *Mumford-Shah* (**30 points**): Implement Mumford-Shah approximation using Ambrosio-Tortorelli, Notes 8, Eq. (2). Perform the segmentation on the following images. Show u and v. Decide how a curve C can be approximated from v, show C approximation. Explain how to change the parameters with respect to the standard deviation of the noise. The images will be posted on the course website.

   (a) Image 1

(b) Image 1 with additive white Gaussian noise, 3 instances: $\sigma = 5, 10, 20$.

(c) Image 2.

2. *Optical Flow* (**40 points**): Implement a simplified model of Brox et al functional for optical flow, Notes 9, Eq. (6), (7), (8)) for only 2 images (no derivative in t) and for $\gamma = 0$ (no second term in Eq. (7)). Read the paper (in the exercise files) to get the relevant E-L equations for your energy. Perform optical flow on the following 2 sequences. Show the flow and analyze the results. You can use Matlab command "quiver" to show the flow direction, check that arrows are actually seen in you doc (or sparsify the arrow-image). See added explanation on pyramid implementation (with the image files).

(a) Image-Set 3 (synthetic).

(b) Image-Set 4 (real example).