

Exercise 2: TV Denoising and Deblurring

Exercise due on 1.1.2018

Please send a PDF file containing a full solution (analytic and experimental results) and the Matlab files folder to **Tal Feld, stmfeld@gmail.com**. The subject of the mail should be **"Exercise 2"**. In the mail please write the full names + ID of the participants.

1 Analytic exercises (30 points)

1. Euler-Lagrange (15 points).

- (a) E-L of higher derivative. Find the E-L equation in 1D of

$$F(u) = \int_{\Omega} (u_{xx})^2 dx.$$

Use the basic Gateaux derivative definition (given in Notes 4) to compute it.

- (b) Prove the E-L of the deconvolution fidelity term, as it appears for instance in Notes 4, Section 2.5.2. That is, $\partial_u F(u, f, H)$, where $F(u, f, H) = \|f - u * H\|_{L^2}^2$.

2. Relation to non-linear diffusion (15 points).

- (a) Verify that the gradient descent of the functional $J_{PM}(u)$ on Notes 4, p. 4 bottom, yields the Perona-Malik equation (p. 5 top).
- (b) Compute $J_{PM}(u)$ in the 1D case, $\Omega = [-1, 1]$, where u is a unit step function centered at $x = 0$, that is $u = 0$ for $-1 \leq x < 0$ and $u = 1$ for $0 \leq x \leq 1$. One can use any reasonable approximation

for the step. For instance, a partition of the step into 2 constant regions and a linear slope of width h between them (taking $h \rightarrow 0$).

- (c) What would be $J_{PM}(u)$ for a piece-wise constant 1D signal? (no full proof is needed, just a short explanation of your claim in words).
- (d) Explain these results and their implications (possible artifacts, need for additional regularization etc.).

2 Matlab experiments (70 points)

Use the theory, algorithms and numerical schemes described in Notes 7 for TV denoising and deconvolution. In the first 2 questions - choose your own images. In Q-3 (Blind deconv.) - use the images given on the website.

1. *TV Denoising (20 points)*: Add 2 kinds of noise: White Gaussian noise with standard deviation 10 (images scaled to $[0..255]$). Uniform noise between $[-15, 15]$.
 - (a) Implement explicit scheme with $\varepsilon = 1$ and denoise.
 - (b) Compare the behavior for the two types of noise.
2. *TV Deconvolution (20 points)*: Implement TV deconvolution using explicit scheme or lagged diffusivity.
 - (a) 1D - blur a step, add low noise and high noise. Deconvolve.
 - (b) 2D - repeat for a blurred image with some noise.
 - (c) Try with various parameters. Explain the relation between λ and the level of noise.
3. *Blind TV Deconvolution (30 points)*: Creative part. Design an algorithm that tries to solve heuristically a blind deconvolution problem. Any justified procedure / algorithm is welcome.
 - (a) Try the algorithm on the first image given in the course website (convolved synthetically).
 - (b) Try the algorithm on the second image given in the course website (naturally blurred).

Try various parameters, explain your results, what are your conclusions and insights?