Course 049064: Variational Methods in Image Processing

Exercise 2: TV Denoising and Deblurring

Exercise due on 26.12.2016

Please send a PDF file containing a full solution (analytic and experimental results) and the Matlab files folder to Guy Gilboa, guy.gilboa@ee.technion.ac.il. The subject of the mail should be **"Exercise 2"**. In the mail please write the full names + ID of the participants.

1 Analytic exercises (30 points)

1. **Euler-Lagrange (15 points).**

   (a) E-L of higher derivative. Find the E-L equation in 1D of

   $$F(u) = \int_\Omega |u_{xx}| dx.$$

   Use the basic Gateaux derivative definition (given in Notes 4) to compute it. To get the intuition, you can start with the quadratic form.

   (b) Prove the E-L of the deconvolution fidelity term, as it appears for instance in Notes 4, Section 2.5.2. That is, $\partial_u F(u, f, H)$, where $F(u, f, H) = \|f - u * H\|^2_{L^2}$.

2. **Relation to non-linear diffusion (15 points).**

   (a) Verify that the gradient descent of the functional $J_{PM}(u)$ on Notes 4, p. 4 bottom, yields the Perona-Malik equation (p. 5 top).

(b) Compute $J_{PM}(u)$ in the 1D case, $\Omega = [-1, 1]$, where u is a unit step function centered at $x = 0$, that is $u = 0$ for $-1 \leq x < 0$ and $u = 1$ for $0 \leq x \leq 1$. One can use any reasonable approximation for the step. For instance, a partition of the step into 2 constant regions and a linear slope of width h between them (taking $h \to 0$).

(c) What would be $J_{PM}(u)$ for a piece-wise constant 1D signal? (no full proof is needed, just a short explanation of your claim in words).

(d) If one convolves the gradient in the computation of the P-M diffusion coefficient $\tilde{C}_{PM} = 1/(1 + (|g_\sigma * \nabla u|/k)^2)$, what would be the effect on the process?

2 Matlab experiments (70 points)

Use the theory, algorithms and numerical schemes described in Notes 5 for TV denoising and deconvolution. In the first 2 questions - choose your own images. In Q-3 (Blind deconv.) - use the images given on the website.

1. *TV Denoising* **(20 points)**: Add 2 kinds of noise: White Gaussian noise with standard deviation 8 (images scaled to [0..255]). Uniform noise between $[-10, 10]$.

   (a) Implement explicit scheme with $\varepsilon = 1$ and denoise.

   (b) Compare the behavior for the two types of noise.

2. *TV Deconvolution* **(20 points)**: Implement TV deconvolution using explicit scheme or lagged diffusivity.

   (a) 1D - blur a step, add low noise and high noise. Deconvolve.

   (b) 2D - repeat for a blurred image with some noise.

   (c) Try with various parameters. Explain the relation between λ and the level of noise.

3. *Blind TV Deconvolution* **(30 points)**: Creative part. Design an algorithm that tries to solve heuristically a blind deconvolution problem. Any justified procedure / algorithm is welcome. If your suggestion does not work well, you may try implementing the Chan-Wong blind deconvolution algorithm (Notes 5, Section 3), which has its difficulties.

   (a) Try the algorithm on the first image given in the course website (convolved synthetically).

(b) Try the algorithm on the second image given in the course website (naturally blurred).

Try various parameters, explain your results, what are your conclusions and insights?