The 1931 CIE standard observer (2°)

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Spectral tristimulus values of equal-energy spectrum</th>
<th>Wavelength (nm)</th>
<th>Spectral tristimulus values of equal-energy spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mathrm{X} )</td>
<td>( \mathrm{Y} )</td>
<td>( \mathrm{Z} )</td>
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<tr>
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<td>0.0014</td>
<td>0.0000</td>
<td>0.0005</td>
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<tr>
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<td>0.0225</td>
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<td>0.0014</td>
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<tr>
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<td>0.0050</td>
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<td>590</td>
<td>0.3618</td>
<td>0.0056</td>
<td>0.0052</td>
</tr>
</tbody>
</table>

**Totals:** | 21.3714 | 21.3714 | 21.3715

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The diagram shows the spectral tristimulus values for different wavelengths. The X, Y, and Z values correspond to the primary stimuli required to match an equal-energy spectrum.

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C1
Chromaticity diagram based on the CIE standard observer and coordinate for colorimetry.

Suggested definitions of color designations for self luminous sources.
For spectral colours \( f(\lambda) = \delta(\lambda - \lambda_0) \), thus giving us the previously stated result. In general we don't need the absolute values \( \overline{X}, \overline{Y} \) and \( \overline{Z} \) as we are interested in the relation between them thus we can normalize them, giving tri-chromaticity

\[
\begin{align*}
x &= \frac{\overline{X}}{\overline{X} + \overline{Y} + \overline{Z}} \\
y &= \frac{\overline{Y}}{\overline{X} + \overline{Y} + \overline{Z}} \\
z &= \frac{\overline{Z}}{\overline{X} + \overline{Y} + \overline{Z}}
\end{align*}
\]

Thus every distribution can be expressed in terms of \( x, y \) and \( z \), except for intensity measured in photometry.

Since \( x + y + z = 1 \), \( x \) and \( y \) are sufficient to represent the colour. This representation is used in the chromaticity diagram (see page 9b).

### 3.3.1. Characteristics of the Chromaticity Diagram

- All colours are found under the curve \( x + y + z = 1 \), i.e. under \( x+y=1 \) in the diagram.
- The diagram also contains impossible combinations, e.g. \( x = 0 \) and \( y = 1 \) is not a real colour.
- The spectral colours map on to a continuous curve.
- If the wavelength is greater than 780nm or less than 380nm its tri stimulus does not appear in the diagram, since \( x = y = z = 0 \).
- Colours that can be mapped are called "real colours" while the rest are called "non real colours". Every point in the "tongue" of the diagram represents a different colour.

### 3.3.2. Colour Combinations in the Chromaticity Diagram

The linear characteristic of the diagram allows us to display all the colours lying on the line between two sources. If three sources are used then all colours in the triangle can be created by an appropriate combination of the sources.

What happens if we try to display a colour from outside the triangle? We can display R using a combination of S and P and we can also match this using a combination of M and N. We can create any colour in the triangle from the three sources N, M and P by using the appropriate luminous flux. This concept is used in colour TV.
3.3.3. Dominant Wavelength (hue) and Purity (saturation)

The dominant wavelength is the wavelength of the spectral colour which in combination with white produces the desired colour. E.g. \( \lambda_m \) for the colour M (Diagram \( \varphi \)).

Types of white light:

1) \( x = y = z = 1/3 \) - notation: E or W
2) Black body radiation at \( T = 2850^\circ K \): A (0.4075,0.4475)
3) Black body radiation at \( T = 4880^\circ K \): B (0.3485,0.3518)
4) Black body radiation at \( T = 6740^\circ K \): C (0.3163,0.3101)

2,3 and 4 are arbitrary definitions and may vary from reference to reference.

White \( \mathbb{W} \) is defined by E or W = Equal or White.

The temperature of the colour correlates to the black body radiation temperature. The "hotter" the body the closer it is to blueish white.

Colours found in the dotted triangle can not be achieved using a combination of spectral colours. We draw a line in the opposite direction and get \((-\lambda_m\) negative) written as \(\lambda_m\).

Saturation describes the distance of a given colour from white. (The closer to white the lower the saturation). It is a quantitative measurement. For normal colours eg. at M we get: :-

\[ 0 \leq \frac{|MW|}{|PW|} \leq 1 \]

For colours in the dotted region eg. Q we get :-

\[ 0 \leq \frac{|QW|}{|NW|} \leq 1 \]

A large saturation value means that the colour is close to a spectral colour.

3.3.4. Complementary Colours

This is the colour which when combined with given colour results in white. It is found on the line which passes through the given colour and white.
3.3.5 Representation of Colours using Hue, Saturation and Brightness (Intensity) Coordinates - (HSI)

r - Saturation or Purity.
z - Brightness or Luminance.
y - Hue or Dominant wavelength.

3.4 Subtractive Process

This is the method for achieving colours in printing and copying.

In every pass through a filter the white light "loses" the wavelengths not transmitted by the filter. The colours must be transparent to allow transmission to other filters. The three accepted filters are as follows:

Filter 1

Filter 2

Filter 3

By manipulating the densities of these three filters we can "play" with the primaries. If all the colours are sparse we get white. If they are maximally dense we get black. In practice it is difficult to achieve black thus an additional black layer is printed.

Cyan + Red = White
Yellow + Blue = White
Magenta + Green = White

The three colours CMY are the complement of RGB. This concept is used in both colour copying and the creation of colour negatives
Creation of a negative

In the development phase we colour the coloured layers that effect the area in the printing phase. White light passes through the negative and reacts according to the layers in the negative.

Printing the negative on Paper

The development of the print resembles the negative creation process but results in the original colours.

Positive Film

The dotted areas do not influence the light passing through, thus in order to get red we must filter cyan which transmits the red component of the original white light.
4. **SPATIAL FREQUENCY**

The visual signal is based on changes in luminance (and colour) dependant on position.

\[ E(x, y) \]

\[ x \]

4.1. **Contrast**

The spatial frequency is determined by the number of cycles per unit length. Contrast is given by :-

\[ C = \frac{(B_{\text{max}} - B_{\text{min}})}{(B_{\text{max}} + B_{\text{min}})} \]

Each axis is considered separately giving us \( w_s \) and \( w_r \). Contrast perception is dependant on :

* \( C \)
* \( w_s \) and \( w_r \) \( [\text{c.p.d}] \) - Cycles Per Degree

![Contrast Sensitivity](image)

\[ \text{fin}(x,y) \rightarrow \text{MTF} \cdot h(x,y) \rightarrow \text{fout}(x,y) \]

\[ \text{fout}(x,y) = \text{fin}(x,y) \cdot h(x,y) = \int \int \text{fin}(x',y') \cdot h(x-x',y-y') \cdot dx'dy' \]

\[ \text{Fout}(u,v) = \text{Fin}(u,v) \cdot (H(u,v)) \]

4.2. **Fourier Transform**

The 2-D Fourier Transform is given by :-

\[ F(u,v) = \int \int f(x,y) \cdot e^{-2\pi i (ux + vy)} dx dy \]

\[ f(x,y) = (1/4\pi^2) \int \int F(u,v) \cdot e^{2\pi i (ux + vy)} dudv \]

**Characteristics:**
Real \( f(x,y) \) \( \Rightarrow \) \( F(u,v) = F(-u, -v) \)

Shift \( f(x-x_0, y-y_0) \) \( \Rightarrow \) \( F(u, v)e^{j2\pi (ux_0 + vy_0)} \)

Scalar \( f(ax, by) \) \( \Rightarrow \) \( F(u, v)|a|b \)

Linearity \( f(x,y) + g(x,y) \) \( \Rightarrow \) \( F(u,v) + G(u,v) \)

Separability \( f(x) \Leftrightarrow F(u) \& g(y) \Leftrightarrow G(v) \)

Then \( f(x)g(y) \Leftrightarrow F(u)G(v) \)

Power Eqn: \( \int \int |f(x,y)|^2 \, dx \, dy = (1/4\pi^2) \int \int |F(u,v)|^2 \, du \, dv \)

4.3. Filters in the Frequency Plane

For the 2-D case we get:

LPF \( \triangleright \) detects continuous areas.
HPF \( \triangleright \) detects contrast but no dc.
BPF \( \triangleright \) closest resemblance to the vision system.

4.4. Hankel Transform

This is used in cases where we find radial symmetry.

\( f(x,y) = \tilde{f}(r); \quad r^2 = x^2 + y^2 \)

\( F(u,v) = \tilde{F}(q); \quad q^2 = u^2 + v^2 \)

where \( \tilde{F}(u,v) \equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi (ux + vy)} \, dx \, dy \).